

LBL-38110  
 UCB-PTH-95/45  
 SLAC-PUB-95-7089  
 hep-ph/9601295  
 January 1996

## Extracting $R_b$ and $R_c$ Without Flavor Tagging

Jonathan L. Feng<sup>\*†</sup> and Hitoshi Murayama<sup>\*</sup>

*Department of Physics, University of California, Berkeley, California 94720*

*and*

*Theoretical Physics Group, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*

James D. Wells<sup>‡</sup>

*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309*

### Abstract

At present, two outstanding discrepancies between experiment and the standard model are the measurements of the hadronic branching fractions  $R_b$  and  $R_c$ . We note that an independent measurement of these branching fractions may be obtained from the width of hadronic  $Z$  decays with a prompt photon,  $\Gamma_{q\bar{q}\gamma}$ , along with the total hadronic decay rate,  $\Gamma_{\text{had}}$ , and an additional theoretical assumption. Such an analysis requires no flavor tagging. We consider several plausible theoretical assumptions and find that the current value of  $\Gamma_{q\bar{q}\gamma}$  favors larger  $R_b$  and smaller  $R_c$  relative to standard model predictions, in accord with the direct measurements. If  $\Gamma_{q\bar{q}\gamma}$  and  $\Gamma_{\text{had}}$  are combined with the direct measurements, generation-blind corrections to all up-type and all down-type quark widths are most favored. An updated measurement of  $\Gamma_{q\bar{q}\gamma}$  with the currently available LEP data is likely to provide an even stronger constraint on both the branching fraction discrepancies and their possible non-standard model sources.

---

<sup>\*</sup>Work supported in part by the Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

<sup>†</sup>Research Fellow, Miller Institute for Basic Research in Science.

<sup>‡</sup>Work supported by the Department of Energy under Contract DE-AC03-76SF00515.

The LEP and SLC  $e^+e^-$  colliders have provided many impressive confirmations of the standard model (SM) through high-precision studies of the  $Z$  boson. At present, however, the combined average of direct measurements of the  $Z$  branching fractions  $R_b \equiv \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$  and  $R_c$  disagree with SM predictions at the level of  $3.7\sigma$  and  $2.3\sigma$ , respectively [1,2]. These direct measurements rely heavily on flavor tagging. It is therefore essential that the flavor tagging efficiencies be calibrated accurately. Impressive techniques have recently been developed, including, most notably, the double-tag method for calibrating  $b$ -tagging efficiency, which is limited basically by statistics only. However, given their status as two of the most significant deviations from the SM, it is worth investigating alternative methods for measuring  $R_b$  and  $R_c$  that are independent of the systematic uncertainties inherent in the direct measurements.

A measurement of the decay width of prompt photon production in hadronic  $Z$  decays, which we denote  $\Gamma_{q\bar{q}\gamma} \equiv \Gamma(Z \rightarrow q\bar{q}\gamma)$ , provides such an alternative. The total hadronic decay width is  $\Gamma_{\text{had}} = \sum_{i=u,c,d,s,b} \Gamma_i$ . In the width  $\Gamma_{q\bar{q}\gamma}$ , however, the up-type quark contribution is enhanced, and so  $\Gamma_{q\bar{q}\gamma} \propto 4 \sum_{i=u,c} \Gamma_i + \sum_{i=d,s,b} \Gamma_i$ . These two measurements, then, along with an assumption relating the light quark widths to those of  $b$  and  $c$ , provide flavor tagging independent determinations of  $R_b$  and  $R_c$ . They may also provide additional constraints on possible deviations from SM values.

By definition, events contributing to  $\Gamma_{q\bar{q}\gamma}$  are events in which the photon is radiated from a primary quark, *i.e.*, one of the two quarks that couples directly to the  $Z$ . The uncertainties in  $\Gamma_{q\bar{q}\gamma}$  arise from backgrounds where an isolated photon comes from other sources, *e.g.*, initial state radiation and hadronization, and also from difficulties in the Monte Carlo modeling [3]. A global average of results from currently available analyses [4] gives  $R_{q\bar{q}\gamma} \equiv \Gamma_{q\bar{q}\gamma}^{\text{SM}}/\Gamma_{q\bar{q}\gamma} = 1.077 \pm 0.042$  (exp.)  $\pm 0.04$  (th.) [5]. (Note that  $R_{q\bar{q}\gamma}$  is defined, following Ref. [5], as the theoretical value divided by the experimental value.) It is interesting to note that the current central value of  $\Gamma_{q\bar{q}\gamma}$  is about  $1.3\sigma$  below the SM prediction.

Given the currently available event sample of  $\sim \mathcal{O}(10^7)$  hadronic  $Z$  events, the statistical error may be reduced to  $\sim 1\%$  [6]. The overall error would then be dominated by systematic errors, which are primarily uncertainties in parton shower modeling and  $\alpha_s$  and have been estimated to be  $\sim 3.5\%$  [6]. The total fractional error of  $\Gamma_{q\bar{q}\gamma}$  may therefore be improved from  $5.8\%$  to  $\sim 3.7\%$  after all the LEP data is analyzed [7]. Such an updated experimental analysis will increase the power of this study considerably, as will be seen below.

To determine  $R_b$  and  $R_c$  from the two measurements  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$ , it is clear that we must choose a theoretically motivated framework for discussing deviations from SM branching fractions. We begin by parametrizing possible shifts in the partial widths by the fractional deviations  $\delta_q$ , defined by

$$\Gamma_q = \Gamma_q^{\text{SM}} (1 + \delta_q) , \quad (1)$$

where  $\Gamma_q$  is the partial width  $\Gamma(Z \rightarrow q\bar{q})$ , and  $\Gamma_q^{\text{SM}}$  is its SM value. With this definition, the shifts in the observables we will analyze are

$$\delta R_i = \frac{\Gamma_i^{\text{SM}} \delta_i - R_i^{\text{SM}} \sum_q \Gamma_q^{\text{SM}} \delta_q}{\Gamma_{\text{had}}^{\text{SM}} + \sum_q \Gamma_q^{\text{SM}} \delta_q}, \quad (2)$$

$$\delta \Gamma_{\text{had}} = \sum_q \Gamma_q^{\text{SM}} \delta_q, \quad (3)$$

$$\delta \Gamma_{q\bar{q}\gamma} \propto 4 \sum_{f=u,c} \Gamma_f^{\text{SM}} \delta_f + \sum_{f=d,s,b} \Gamma_f^{\text{SM}} \delta_f, \quad (4)$$

where  $q = u, c, d, s, b$ , and  $i = b, c$ .

The above parametrization accommodates a variety of new physics sources, such as  $Z$ - $Z'$  mixing, new oblique corrections, and  $Zq\bar{q}$  vertex corrections. Implicit in Eq. (4), however, is the assumption that the effects of new physics on the prompt photon width are proportional to the primary quark charges, as is true when the photon is radiated from a primary quark. In general, this may be violated, for example, by box diagrams in which the photon is attached to an internal loop. We assume, however, that the effects of such diagrams are smaller than those of oblique and vertex corrections, as is typically true in many new physics scenarios [8].

At this stage, we have parametrized deviations from the SM in the five parameters  $\delta_q$ . To extract  $R_b$  and  $R_c$  from  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$ , we must further reduce the number of parameters to two. We consider the following scenarios, where the listed  $\delta_q$  parameters are allowed to vary subject to the given constraints, and *all unlisted  $\delta_q$ 's are assumed to vanish*:

- (I)  $\delta_c, \delta_b$  ( $c/b$  case)
- (II)  $\delta_u = \delta_c, \delta_d = \delta_s = \delta_b$  (generation-blind case)
- (III)  $\delta_u = \delta_c, \delta_b$  ( $uc/b$  case)
- (IV)  $\delta_b$  ( $b$  case) .

These scenarios are by no means exhaustive, but have a number of interesting motivations. The  $c/b$  case is an obvious first choice, as it is the most conservative scenario consistent with the anomalous direct measurements of  $R_c$  and  $R_b$ . One should note, however, that  $\delta_u \neq \delta_c$  and  $\delta_d \neq \delta_s$  are each theoretically disfavored by the constraints from flavor-changing neutral currents (FCNC). Suppose the  $Zc\bar{c}$  and  $Zu\bar{u}$  couplings differ by  $\epsilon \simeq 5\%$ , as required to achieve a 10% reduction in  $R_c$ . Suppose also that the mass eigenstates  $u$  and  $c$  are rotated by an angle  $\theta$  relative to the interaction eigenstates. Let us consider the states  $u_L$  and  $c_L$ . The rotation generates the FCNC vertex  $g_Z^u \epsilon \theta Z_\mu (\bar{u}_L \gamma^\mu c_L) + \text{c.c.}$ , where  $g_Z^u \equiv e \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) / \sin \theta_W \cos \theta_W$ , and  $\theta_W$  is the weak mixing angle.  $Z$  boson exchange then generates a four-fermion operator  $\frac{1}{2} (g_Z^u \epsilon \theta / m_Z)^2 \bar{u}_L \gamma^\mu c_L \bar{u}_L \gamma_\mu c_L$ , which contributes to  $D^0 - \bar{D}^0$  mixing. From the experimental bound  $\Delta m_D < 1.3 \times 10^{-13}$  GeV, one obtains a rough bound  $\epsilon \theta \lesssim 3 \times 10^{-4}$ , or  $\theta \lesssim 6 \times 10^{-3}$  with  $\epsilon = 0.05$ , where we have taken  $f_D^2 B_D \simeq (300 \text{ MeV})^2$ . A difference in  $\delta_d$  and  $\delta_s$  is similarly constrained by  $K^0 - \bar{K}^0$  mixing. Simultaneous deviations from both  $\delta_u \simeq \delta_c$  and  $\delta_d \simeq \delta_s$  are excluded. These arguments do not completely exclude the possibility of *either*  $\delta_u \neq \delta_c$  *or*  $\delta_d \neq \delta_s$ . However, we see that, without some additional symmetries, such possibilities require fine-tuning, and are therefore unnatural and theoretically disfavored.

We are therefore led to consider scenarios with  $\delta_u = \delta_c$  and  $\delta_d = \delta_s$ . The generation-blind case listed above is perhaps the most well-motivated. For example, a mixing between  $Z$  and a  $Z'$  boson whose coupling is generation-blind leads to this case, as do flavor-independent vertex corrections. In addition, oblique corrections depend only on quantum numbers, and so a scenario in which oblique corrections are the dominant effect of new physics is an example of the generation-blind case. The  $uc/b$  scenario is the most conservative scenario that is consistent with both the LEP direct measurements of  $R_b$  and  $R_c$  and the theoretical considerations of the previous paragraph.

Finally, one can also consider the measured discrepancy in  $R_c$  to be a large statistical fluctuation and allow only  $\delta_b$  to be non-vanishing. This scenario, the  $b$  case, is realized if there is a gauge boson that couples only to the third generation and mixes with the  $Z$  boson [9], or a large vertex correction to the  $Zb\bar{b}$  vertex from superparticles [10] or technicolor [11]. This possibility could help resolve the longstanding difference between  $\alpha_s(m_Z^2) = 0.123$  extracted from the  $Z$  lineshape [1] (in the SM) and the lower  $\alpha_s(m_Z^2) \simeq 0.110$  from many low energy observables [12]. In fact, the change in  $\alpha_s$  for a given shift in the electroweak contribution to  $\Gamma(Z \rightarrow b\bar{b})$  is  $\delta\alpha_s \simeq -0.7\delta_b$ . Using  $R_b^{\text{expt}} = 0.2205 \pm 0.0016$  when  $R_c$  is fixed to its SM value [1], a shift  $\delta_b \simeq 0.02$  makes the measured and SM predictions of  $R_b$  consistent to about  $1\sigma$  and simultaneously brings the value of  $\alpha_s$  extracted from the  $Z$  lineshape down to about 0.110.

For each of these scenarios, we now use the measured values of  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$  to determine  $R_b$  and  $R_c$ , and we compare the extracted values of these branching fractions to the direct measurements. Table I shows the measured values and SM predictions for  $R_b$ ,  $R_c$ ,  $\Gamma_{\text{had}}$ , and  $R_{q\bar{q}\gamma}$  [2]. In applying the measured values of these quantities to constrain the various scenarios, we assume that the new physics does not significantly alter the detection efficiency of the prompt photon signal. If it does, the parameters  $\delta_q$  and the efficiency are correlated, which complicates the analysis. However, as noted above, we assume that oblique or vertex corrections are the dominant effects of new physics in this analysis. These corrections preserve all kinematical distributions of the jets and photon for each quark chirality, and the efficiency is therefore insensitive to the new physics effects.

The error for each of the observables is determined by adding in quadrature the experimental measurement error and the uncertainties in the top quark mass and strong coupling constant, which we take to be  $m_t = 175 \pm 15$  GeV and  $\alpha_s(m_Z^2) = 0.118 \pm 0.006$ . Note that we cannot use the value of  $\alpha_s$  extracted from the global fit, because we allow deviations of the widths  $\Gamma_q$  from the SM. The  $\alpha_s$  measurements from low-energy data and jet shape variables do not rely on electroweak physics, and so may be used in this analysis.

We present our results in Fig. 1 for the extracted values of  $R_b$  and  $R_c$  for each of the first three theoretical assumptions discussed above. (The  $b$  case will be discussed below.) For each scenario, the measured values of  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$  determine a preferred region of the  $(R_b, R_c)$  plane. The  $1\sigma$  contours are plotted in Fig. 1. All regions are long and narrow.

The width of each region is determined by  $\Gamma_{\text{had}}$ , which is tightly constrained relative to the other measurements, and the parametrization of the particular theoretical scenario. For example, in the generation-blind case, no variation in the  $\delta_q$  parameters changes  $R_b$  without changing  $R_c$ , so the associated band is very thin. The slopes vary from case to case because  $\Gamma_{\text{had}}$  constrains different linear combinations of  $R_b$  and  $R_c$  in the different scenarios. The positions of the regions are determined by the overlap of the  $\Gamma_{q\bar{q}\gamma}$  band with the  $\Gamma_{\text{had}}$  band. The lengths are different for each case because the relative angle between the two bands varies; if they are more parallel, the overlap region is longer.

There are a number of interesting features of Fig. 1. First of all, it is noteworthy that the SM values for  $R_b$  and  $R_c$  are outside the  $1\sigma$  region for all scenarios. This is a reflection of the fact that the measured value of  $\Gamma_{q\bar{q}\gamma}$  currently differs from the SM prediction by  $1.3\sigma$ . Second, for these theoretical assumptions, the  $1\sigma$  contours prefer higher  $R_b$  and lower  $R_c$  than the SM values, because the measured  $\Gamma_{q\bar{q}\gamma}$  is below the SM prediction. Since the error in  $\Gamma_{q\bar{q}\gamma}$  is much larger than that of  $\Gamma_{\text{had}}$ , the lengths of the regions scale as the error in  $\Gamma_{q\bar{q}\gamma}$ , and it is easy to see how the regions would shrink as the accuracy in  $\Gamma_{q\bar{q}\gamma}$  improves. If the error reduces to 3.7% as expected given the currently available LEP statistics discussed above, the lengths of the regions will decrease by a factor of 0.64. Depending on where the central value falls, the measurement of  $\Gamma_{q\bar{q}\gamma}$  may be quite significant. For example, if the central value were to remain at its present value, the  $\Gamma_{q\bar{q}\gamma}$  measurement would disagree with the SM at the level of  $2.1\sigma$ .

The  $b$  case must be discussed separately since it has only one free parameter. What is interesting in this case is that one can extract  $R_b$  from  $\Gamma_{\text{had}}$  alone, or from  $\Gamma_{q\bar{q}\gamma}$  alone. These two extracted values can then be compared to check the consistency of the scenario. From  $\Gamma_{\text{had}}$  we obtain  $R_b = 0.2170 \pm 0.0015$  ( $0.2191 \pm 0.0015$ ) for  $\alpha_s(m_Z) = 0.118$  (0.110). On the other hand,  $\Gamma_{q\bar{q}\gamma}$  gives  $R_b = 0.0877^{+0.1016}_{-0.0877}$ . The extracted values of  $R_b$  differ by about  $1.3\sigma$ , irrespective of the value assumed for  $\alpha_s$ . A future improvement on  $\Gamma_{q\bar{q}\gamma}$  will certainly strengthen our ability to determine this scenario's consistency.

We have also plotted in Fig. 1 the 68% and 95% C.L. contours for the direct measurements. The combined measurement of all four observables provides an opportunity to differentiate various new physics scenarios. For example, it is evident from Fig. 1 that the direct measurements of  $R_b$  and  $R_c$  are most consistent with those extracted from  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$  in the generation-blind case. To quantify such a discussion, we now turn to the results of global fits to all four observables for each of the cases. In the global fits, we treat the errors in  $m_t$  and  $\alpha_s$  as intrinsic uncertainties as before. Alternatively, we could allow  $m_t$  and  $\alpha_s$  to vary in the fits, but we choose to regard them as uncertainties to simplify the discussion. The correlation of  $R_b$  and  $R_c$  in the direct measurements is also included.

For the  $c/b$ , generation-blind, and  $uc/b$  cases, we find that the minimum  $\chi^2/\text{d.o.f.}$  is 4.0/2, 1.6/2, and 5.1/2, respectively. We find that the generation-blind case has no difficulty describing the data, while the other cases are disfavored at more than 85% C.L. Indeed, it was

shown that a mixing of  $Z$  with an extra  $E_6$   $U(1)$  gauge boson could improve the consistency between theory and data [13]. Unfortunately, this particular realization of the generation-blind case fails in the lepton sector, and such an interpretation is excluded. Nonetheless, our analysis clearly shows that as a description of the hadronic widths and branching fractions, the generation-blind case is the most favored of the new physics scenarios we have considered. For the  $b$  case, there are two possible attitudes. If we fix  $R_c$  at its SM predicted value and take  $\alpha_s = 0.118$ , a fit to  $\Gamma_{\text{had}}$ ,  $\Gamma_{q\bar{q}\gamma}$  and  $R_b^{\text{expt}}$  has a minimum  $\chi^2/\text{d.o.f.}$  of 4.0/2. If we use the correlated experimental values for both  $R_c$  and  $R_b$ , the minimum is 8.3/3. However if we take  $\alpha_s = 0.110$ , as advocated by Ref. [12], the minimum  $\chi^2/\text{d.o.f.}$  values for the two methods improve to 2.3/2 and 6.5/3, respectively.

Finally, we note that, imposing only the naturalness condition from the FCNC considerations discussed above, the most general scenario allows all  $\delta_q$  parameters to vary subject to the constraints

$$(V) \quad \delta_u \simeq \delta_c, \delta_d \simeq \delta_s, \delta_b \quad .$$

This case is relevant if both large generation independent  $\delta_q$  shifts, *e.g.*, shifts resulting from large non-standard oblique corrections, and large  $Zb\bar{b}$  specific corrections are present. An analysis of such a case, however, is beyond the scope of this letter.

In conclusion, we find that the measurements of  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$ , when combined with a theoretical assumption, provide a significant constraint on quark partial widths without relying on flavor tagging. In light of FCNC constraints, four plausible theoretical assumptions were considered. For each case, we extracted  $R_b$  and  $R_c$  from  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$  and determined favored regions in the  $(R_b, R_c)$  plane. The current measurement of  $\Gamma_{q\bar{q}\gamma}$  prefers larger  $R_b$  and smaller  $R_c$  relative to SM predictions. These regions, when compared with the direct determinations of  $R_b$  and  $R_c$ , may be used to help select among the many possible models of physics beyond the SM. Of the four examples presented above, it appears that generation-blind corrections provide a good fit to the data. Scenarios in which only the  $b$  and  $c$  quark partial widths are allowed to deviate from their standard model values are disfavored in this analysis. The analysis of all currently available LEP data is expected to reduce the uncertainty in  $\Gamma_{q\bar{q}\gamma}$  and will significantly improve our ability to detect and interpret deviations from the standard model.

## ACKNOWLEDGMENTS

We thank I. Hinchliffe, M. Mangano, P. Mättig, and Y. Shadmi for useful discussions and helpful correspondence.

## REFERENCES

- [1] The LEP Electroweak Working Group, LEPPEWWG-95-02 (1995).
- [2] All SM partial widths are calculated with the aid of ZOPOLE. B. Kniehl, R.G. Stuart, Comput. Phys. Comm. **72**, 175 (1992).
- [3] One potential difficulty lies in the modeling of the fragmentation of quarks (and gluons) into photons, which is flavor-dependent. Such uncertainties may be greatly reduced by imposing isolation cuts to remove most of the fragmentation region. This approach and the errors accompanying the introduction of isolation cuts are discussed in Refs. [4–6].
- [4] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. **C57**, 17 (1993); DELPHI Collaboration, P. Abreu *et al.*, CERN-PPE/95-101; L3 Collaboration, O. Adriani *et al.*, Phys. Lett. **B301**, 136 (1993); OPAL Collaboration, P. D. Acton *et al.*, Z. Phys. **C58**, 405 (1993).
- [5] P. Mättig, invited talk at the International Symposium on Vector Boson Self Interactions, Los Angeles, February 1–3, 1995, CERN-PPE/95-081.
- [6] P. Mättig, H. Spiesberger, and W. Zeuner, Z. Phys. **C60**, 613 (1993).
- [7] It is also possible that the high statistics of the currently available event sample may be used to reduce the systematic errors. However, it is unlikely that the measurement may be improved beyond the 3% level, as measurements of the rate for three-jet events (or, in other words, “prompt gluon production”) suffer from systematic uncertainties in hadronization at this level. See, for example, K. Abe *et al.*, Phys. Rev. Lett. **71**, 2528 (1993).
- [8] Diagrams that violate Eq. (4) are typically suppressed by the effective new physics scale  $\Lambda$ . For example, box diagrams lead to effective contact interactions like  $eF^{\mu\nu}\bar{q}\gamma_\mu qZ_\nu/\Lambda^2$ . In contrast, one may have non-decoupling effects in oblique and vertex corrections. Furthermore, oblique and vertex corrections are enhanced both by internal quark propagators, even after isolation cuts, and by the fact that they are in phase with the corresponding tree-level amplitudes; such enhancements are missing for the other diagrams.
- [9] See, for example, B. Holdom, Phys. Lett. **B351**, 279 (1995). If the mixing shifts  $m_Z$  substantially, it modifies the overall normalization of the decay rates  $G_F m_Z^3$  and the weak mixing angle, resulting in apparent shifts  $\delta_u = \delta_c$ ,  $\delta_d = \delta_s$ , and  $\delta_b$ . This is the most general case naturally compatible with FCNC constraints, case (V).
- [10] A. Djouadi *et al.*, Nucl. Phys. **B349**, 48 (1991); M. Boudware and D. Finnell, Phys. Rev. **D44**, 2054 (1991).
- [11] R. S. Chivukula, S. B. Selipsky, and E. H. Simmons, Phys. Rev. Lett. **69**, 575 (1992).
- [12] M. Shifman, Mod. Phys. Lett. **A10**, 605 (1995); hep-ph/9511469.
- [13] K. Hagiwara, plenary talk presented at the International Symposium on Lepton Photon Interactions, Beijing, August 10–15, 1995, hep-ph/9512425.

# TABLES

TABLE I. Measured and SM values (for  $m_t = 175 \pm 15$  GeV) for four key observables.  $\Gamma_{\text{had}}$  is in GeV. “Pull” is the difference in the measured and SM central values in units of the experimental error.

Observable	Measurement	Standard Model	Pull
$R_b$	$0.2219 \pm 0.0017$	$0.2154 \pm 0.0005$	+3.7
$R_c$	$0.1540 \pm 0.0074$	$0.1711 \pm 0.0002$	−2.3
$\Gamma_{\text{had}}$	$1.7448 \pm 0.0030$	$1.7405 \pm 0.0039$	+1.4
$R_{q\bar{q}\gamma}$	$1.077 \pm 0.058$	1	+1.3



# FIGURES

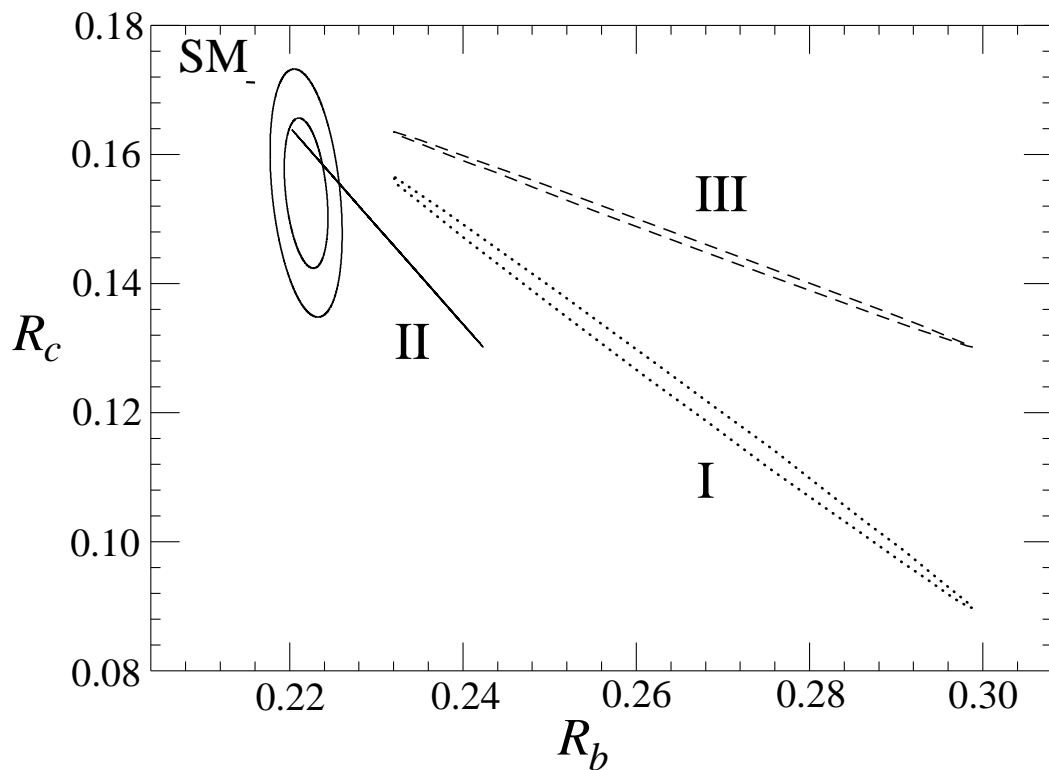


FIG. 1. The  $1\sigma$  allowed regions in the  $(R_b, R_c)$  plane extracted from the measured values of  $\Gamma_{\text{had}}$  and  $\Gamma_{q\bar{q}\gamma}$  in the three scenarios: (I)  $c/b$  case (dotted), (II) generation-blind case (solid), and (III)  $uc/b$  case (dashed). The ellipses are the 68% and 95% C.L. contours for the direct measurements of  $R_b$  and  $R_c$ , and the SM predictions, with  $m_t = 175 \pm 15$  GeV, are given by the very short line segment in the upper-left corner. The current value of  $\Gamma_{q\bar{q}\gamma}$  has an error of 5.8%. This is estimated to improve to 3.7% given the currently available LEP event samples, which will shrink the  $1\sigma$  allowed regions by a factor of 0.64.